

Quantum solution to the Newcomb's paradox

Edward W. Piotrowski

Institute of Theoretical Physics, University of Białystok,
Lipowa 41, Pl 15424 Białystok, Poland

e-mail: ep@alpha.uwb.edu.pl

Jan Śladkowski

Institute of Physics, University of Silesia,
Uniwersytecka 4, Pl 40007 Katowice, Poland

e-mail: sladk@us.edu.pl

Abstract

We show that quantum game theory offers solution to the famous Newcomb's paradox (free will problem). Divine foreknowledge is not necessary for successful completion of the game because quantum theory offers a way to discern human intentions in such way that the human retain her/his free will but cannot profit from changing decision. Possible interpretation in terms of quantum market games is proposed.

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1 Introduction

There is a common belief that the characteristic size of the brain's integral parts is too big to allow for quantum effects being important [1]. But recent experiments show that separated objects of the size of a golf ball can form quantum entangled states even in a room temperature [2]. Physicists successfully apply quantum mechanics to describe a lot of complex system that may have in principle arbitrary size, including black holes or even the

whole Universe. Are there any reasons for quantum modeling of phenomena related to brain activity, consciousness or social behaviour? One can give an answer to this question only after construction and thorough verification of respective models [3]. Below we consider a problem easily susceptible of modeling as a quantum game that should shed some light on the solutions that quantum theory may offer.

In 1960 William Newcomb, a physicist, intrigued the philosopher Robert Nozick with a claim that in an elementary game characterized by the matrix M

$$M := \begin{pmatrix} \$1000 & \$1\,001\,000 \\ 0 & \$1\,000\,000 \end{pmatrix} \quad (1)$$

giving the pay-off of the player 1 in all possible situations, the player 1 is not able to chose his strategy without having any measure of occurring *a posteriori* of any of the four possible events. Rows correspond the player 1's strategies: feminine $|0\rangle_1$ and masculine $|1\rangle_1$ ¹ and columns to opponent's strategies $|0\rangle_2, |1\rangle_2$. It so happens even despite the fact that the feminine strategy dominates the masculine one (that is the pay-off is greater regardless of the opponents strategy). The choice of the masculine strategy $|1\rangle_1$ is more profitable when the event corresponding to the off-diagonal elements of M do not occur and the rest have almost equal probabilities. This might happen if the opponent is able to foresee the player 1 moves. Due to this paradoxical property the above game with indefinite (hidden) set of occurrences became for philosophers, economists and theologians a graceful theme of speculations about free will and its consequences [4, 5]. The disputes, often referred to as newcombmania [6], deserve a thorough analyzis from the quantum game theory point of view [7]-[11]. The development of the probability theory provide us with many intriguing examples where ambiguous specification of the appropriate probability measures resulted in contradiction (Bertrand [13] and Banach-Tarski [14] paradoxes are the most famous ones). One can still find people who regardless of this facts continue philosophical disputes while ignoring the necessity of precise definition of the probabilistic measures in their models. We would like to show that quantum theory may be of help in settling the ambiguities.

¹The use of the adjectives feminine and masculine to underline the character of the strategies will be explained later, see also the Gardner book

2 Quantum description of the game

Quantum game theory exploits the formalism of quantum mechanics in order to offer the players new classes of strategies. Interesting generalization of well known classical games have been put forward [7, 8]. There are arguments that quantum strategies may offer extraordinary tools for biologists [15]–[17]. Economics being the theater of various games and conflicts should not despise these new ideas [18, 12]. We will describe player’s strategies as vectors (often referred to as states) in Hilbert spaces \mathcal{H}_i where the subscripts $i = 1, 2$ distinguish between the player 1 and 2. It is convenient to define the strategy density operator \mathcal{W}

$$\mathcal{W} = \sum_{r,s=1}^2 W_{rs} |r-1\rangle_1 |s-1\rangle_2 {}_1\langle r-1| {}_2\langle s-1|$$

where (W_{rs}) is a matrix with nonnegative entries such that $\sum_{r,s} W_{rs} = 1$ and $|r\rangle_1 |s\rangle_2 {}_1\langle r| {}_2\langle s|$, $r, s \in \{0, 1\}$ are projective operators on the states of the game, $|r\rangle_1 |s\rangle_2 \in \mathcal{H}_1 \otimes \mathcal{H}_2$. For our aims it will be sufficient to use two dimensional Hilbert spaces for the players’ strategies. The states of the classical setting (mixed strategies) are represented by a diagonal matrix (W_{rs}) . Non-diagonal elements of (W_{rs}) describe situations (strategies) that are out of the reach for classical players. Following the classical terminology we will call the pay-off observable \mathcal{M} a Hermitian operator corresponding to the matrix (1):

$$\mathcal{M} := \sum_{r,s=1}^2 M_{rs} |r-1\rangle_1 |s-1\rangle_2 {}_1\langle r-1| {}_2\langle s-1|.$$

Therefore, according to the classical interpretation of the game, the player 1’s expected pay-off is equal to the sum of diagonal elements (trace) of the product of M and the transpose of W :

$$E(\mathcal{M}) := \text{Tr} \mathcal{M} \mathcal{W} = \sum_{rs} M_{rs} W_{rs} = \text{Tr} M W^T.$$

3 Newcomb’s paradox

M. Gardner proposed the following fabulous description of a game with pay-off given by the matrix (1) [4]. An alien Omega (or Alf?) being a omniscient

representative of alien civilization (player 2) offers a human (player 1) a choice between two boxes. The player 1 can take the content of both boxes or only the content of the second one. The first one is transparent and contains \$1000. Omega declares to have put into the second box that is not transparent \$1000000 (strategy $|1\rangle_2$) but only if he foresaw that the player 1 decided to take only the content of that box ($|1\rangle_1$). A male player 1 thinks: *If Omega knows what I am going to do then I have the choice between \$1000 and \$1000000. Therefore I take the \$1000000* (strategy $|1\rangle_1$). A female player 1 thinks: *Its obvious that I want to take the only the content of the second box therefore Omega foresaw it and put the \$1000000 into the box. So the one million dollar is in the second box. Why should I not take more – I take the content of both boxes* (strategy $|0\rangle_1$). The question is whose strategy, male's or female's, is better? One cannot give unambiguous answer to this question without precise definition of the measures of the events relevant for the pay-off.

4 Human's and Omega's strategies

Omega as representative of an advanced alien civilization is certainly aware of quantum properties of the Universe that are still obscure or mysterious to humans. The boxes containing pay-offs are probably coupled. One can suspect this because the human cannot take content of the transparent box only (\$1000). The female player is sceptical about the possibility of realization of the Omega's scenario for the game. She thinks that the choice of the male strategy results in Omega putting the one million dollar in the second box, and after this being done no one can prevent from her taking the content of the both boxes in question (ie \$1001000). But Meyer proposed a quantum tactics [7] that, if adopted by Omega, allows Omega to accomplish his scenario. Let us note that Omega may not be able to foresee the future [4]. For it aims it is sufficient that it is able to discern human intentions regardless of their will or feelings on the matter. The obstacles to this implied by the no-cloning theorem can be overcome by means of teleportation [19]: Omega has must be able to intercept and then return human's strategies. The presented below manipulations leading to thwarting humans are feasible with contemporary technologies. The course of the game may look as follows. At the starting-point, the density operator \mathcal{W} acting on $\mathcal{H}_1 \otimes \mathcal{H}_2$ describes the human's intended strategy and the Omega's strategy based on its prediction

of human's intentions. The actual game must be carried on according to quantum rules that is players are allowed to change the state of the game by unitary action on \mathcal{W} [7, 8]. The human player can only act on her/his q -bit Hilbert space \mathcal{H}_1 . Omega's tactics must not depend on the actual move performed by the human player (it may not be aware of the human strategy): its moves are performed by automatic device that couples the boxes. The Meyer's recipe leads to:

1. Just before the human's move, Omega set the automatic devise according to its knowledge of human's intention. The device executes the tactics $\mathcal{F} \otimes \mathcal{I}$, where \mathcal{I} is the identity transform (Omega cannot change its decision) and \mathcal{F} is the well known Hadamard transform frequently used in quantum algorithms: $F := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.
2. The human player with the probability w uses the female tactics $\mathcal{N} \otimes \mathcal{I}$, where \mathcal{N} is the negation operator² and with the probability $1-w$ the male tactics $\mathcal{I} \otimes \mathcal{I}$.
3. At the final step the boxes are being opened and the built-in coupling mechanism performs once more the transform $\mathcal{F} \otimes \mathcal{I}$ and the game is settled.

5 The course of the game and its result

Let us analyze the evolution of the density operator \mathcal{W} . The players' tactics, by definition, could have resulted in changes in the (sub-)space \mathcal{H}_1 only therefore it suffices to analyze the human's strategies. In a general case the human can use a mixed strategy: the female one with the probability v and the male one with the probability $1-v$. Let us begin with the extreme values of v (pure strategies). If the human decided to use the female strategy ($v=1$) or the male one ($v=0$) then the matrices \mathcal{W}_i , $i = 0, 1$ corresponding to the density operators

$$\mathcal{W}_0 = \sum_{r,s=1}^2 W_{0rs} |r-1\rangle_1 |0\rangle_2 {}_1\langle s-1| {}_2\langle 0|$$

² $\mathcal{N}|0\rangle = |1\rangle$, $\mathcal{N}|1\rangle = |0\rangle$

and

$$\mathcal{W}_1 = \sum_{r,s=1}^2 W_{1rs} |r-1\rangle_1 |1\rangle_{2\ 1} \langle s-1|_2 \langle 1|$$

are calculated as follows:

$$\begin{aligned} \begin{pmatrix} v & 0 \\ 0 & 1-v \end{pmatrix} &\longrightarrow \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v & 0 \\ 0 & 1-v \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 2v-1 \\ 2v-1 & 1 \end{pmatrix} \longrightarrow \\ \frac{w}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2v-1 \\ 2v-1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1-w}{2} \begin{pmatrix} 1 & 2v-1 \\ 2v-1 & 1 \end{pmatrix} &= \frac{1}{2} \begin{pmatrix} 1 & 2v-1 \\ 2v-1 & 1 \end{pmatrix} \longrightarrow \\ \frac{1}{4} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2v-1 \\ 2v-1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} &= \begin{pmatrix} v & 0 \\ 0 & 1-v \end{pmatrix}. \end{aligned}$$

It is obvious that independently of the used tactics, human's strategy takes the starting form. For the mixed strategy the course of the game is described by the density operator

$$\mathcal{W} = v \mathcal{W}_0 + (1-v) \mathcal{W}_1$$

which also has the same diagonal form at the beginning and at the end of the game:

$$\mathcal{W} = v |0\rangle_1 |0\rangle_{2\ 1} \langle 0|_2 \langle 0| + (1-v) |1\rangle_1 |1\rangle_{2\ 1} \langle 1|_2 \langle 1|.$$

Therefore the change of mind resulting from the female strategy cannot lead to any additional profits. If the human using the female tactics (that is changes his/her mind) begins the game with the female strategy then at the end the untransparent box will be empty and he/she will not get the content of the transparent box: the pay-off will be minimal (0). If the human acts just the opposite the transparent box must not be opened but nevertheless the pay-off will be maximal (\$100000). Only if the human begins with the female strategy and then applies the male tactics the content of the transparent box is accessible. If restricted to the classical game theory Omega would have to prevent humans from changing their minds. In the quantum domain the pay-off M_{21} (female strategy and tactics) is possible (the phrase *la donna mobile* gets a quantum context): humans regain their free will but they have to remember that Omega has (quantum) means to prevent humans from

profiting from altering their decisions. In that way quantum approach allows to remove the paradox from the rationally defined dilemma. One can also consider games with more alternatives for the human player. The respective larger pay-off matrices would offer even more sophisticated versions of the Newcomb's observation. But even then there is a quantum protocol that guarantees that Omega keeps its promises (threats) [21].

6 Market interpretation of the game

It is obvious that the above scenario cannot be realized if the actual conditions would differ from Omega's promises. For example, Omega may not be able to predict humans intentions or its understanding of the rules of the game differs from that implied by their expression in human language (cultural differences). There may be much dispute over the question *what Omega really has in mind?* We would like to consider one of the variant that may be interesting in the context of quantum market games [12, 20]. This may result from pondering over the meaning of the term *Omega adopts the same strategy*.

Players in a quantum market game sometimes buy and sometimes sell. A demand representation of the player's strategy is a Fourier transform of his strategy used while supplying the goods [12, 11]. In a simplified model where player's strategies span a finite dimensional Hilbert space we should apply discrete Fourier transform which transforms the demand representation of the strategy, being m -tuple of complex numbers $\langle d|\psi\rangle$ to the supply representation given by

$$\langle s|\psi\rangle = \frac{1}{\sqrt{m}} \sum_{d=0}^{m-1} e^{\frac{2\pi i}{m}sd} \langle d|\psi\rangle.$$

If $m = 2$ then the discrete Fourier transform reduces to the Hadamard transform \mathcal{F} which we have already met. In our case the Hadamard transform switches maximally localized strategies with the the maximally indefinite strategies and vice versa, eg. $\langle d|\psi\rangle = [d=0] \xrightarrow{\mathcal{F}} \langle s|\psi\rangle = \frac{1}{\sqrt{2}}$ (the Iverson notation [22] is used: $[expression]$ denotes the logical value (1 or 0) of the sentence *expression*).

We introduce the nonhomogeneous complex coordinate $z \in \overline{\mathbb{C}}$ to parameterize player's strategies

$$\mathcal{H} \ni |\psi_z\rangle := |0\rangle + z|1\rangle.$$

If the "buying human" decides to use the strategy $|\psi_z\rangle_1$ and the other side of the bargain (Omega) want to play in the same way and therefore uses the supply representation of human's strategy setting its q -bit to $\mathcal{F}(|0\rangle_2 + z|1\rangle_2) = |0\rangle_2 + \frac{1-z}{1+z}|1\rangle_2$ then the quantum state of the game takes the form³

$$\mathcal{W}_z = \frac{(1+z)(1+\bar{z})}{2(1+z\bar{z})^2} (|0\rangle_1 + z|1\rangle_1) ({}_1\langle 0| + \bar{z}{}_1\langle 1|) (|0\rangle_2 + \frac{1-z}{1+z}|1\rangle_2) ({}_2\langle 0| + \frac{1-\bar{z}}{1+\bar{z}}{}_2\langle 1|).$$

Therefore, as in the previous discussion, the female strategy gives not higher a pay-off. The expectation value of the human's pay-off, $\text{Tr} \mathcal{M} \mathcal{W}_z$, is maximal for a superposition of male and female strategies with phase shifted by π (i.e. for $z = -1$). In this case the human is better off than in the previous case but she or he must be cautious because the phase shift by π ($z = 1$) does not change the respective probabilities but result in the lowest expectation value of the pay-off (\$500). Classical human's strategies correspond to $z=0$ (female) and $z = \pm\infty$ (male). The expectation values of the human pay-off with respect to the adopted strategy are presented in Figure 1.

Enthusiasts for newcombmania will certainly find a lot of new quantum solutions to the Newcomb game.

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³Cases when the player may in fact play also "against himself" often happen in market description: demand or supply result from self-consistent strategy of all players. For example in a stock exchange one big bid or transaction can influence the whole market

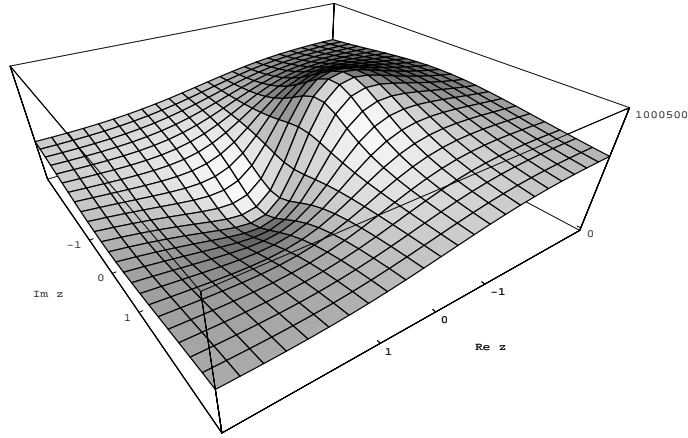


Figure 1: The average human pay-off $\text{Tr}\mathcal{M}\mathcal{W}_z$ in the market version of the Newcomb game.

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